Chapter 27
Vulnerability Assessment for GNSS Constellation Based on AHP-FCE

Bo Qu, Jiaolong Wei, Shuangna Zhang and Liang Bi

Abstract The vulnerability of global navigation satellite system (GNSS) constellation is an important part of the vulnerability of GNSS. The vulnerability assessment for GNSS constellation is helpful to improve GNSS constellation. In this paper, the vulnerability of GNSS constellation is investigated and a vulnerability assessment model for GNSS constellation is suggested, then the vulnerability of GNSS constellation is evaluated by Analytical Hierarchy Process and Fuzzy Comprehensive Evaluation (AHP-FCE). The suggested assessment model concerns about the coverage of GNSS constellation in the situations that 1 satellite, 2 satellites, and 3 satellites are failure, and the mean coverage of GNSS constellation which is affected by satellite failures is used to be the assessment criterions. The vulnerability values of GNSS constellation can be calculated by AHP-FCE according to these criterions. In this paper, the vulnerability of GPS constellation and COMPASS constellation are evaluated, and the assessment results show that GPS constellation is slightly more vulnerable than COMPASS constellation which has more redundant satellites.

Keywords Vulnerability · GNSS constellation · AHP · FCE · Assessment

---

B. Qu (✉) · J. Wei (✉)
The Department of Electronic and Information Engineering,
Huazhong University of Science and Technology, Wuhan, China
e-mail: qb840528@163.com

J. Wei
e-mail: jlwei@mail.hust.edu.cn

S. Zhang · L. Bi
Space Star Technology Co. Ltd, Beijing, China

J. Sun et al. (eds.), *China Satellite Navigation Conference (CSNC) 2013 Proceedings*, Lecture Notes in Electrical Engineering 243,
DOI: 10.1007/978-3-642-37398-5_27,

27.1 Introduction

With the wide application of GNSS on various aspects of national defense and social life, the importance of GNSS is increasing apparently. Therefore, the research on the vulnerability of navigation satellite system is becoming more and more important.

The navigation satellite system typically consists of three segments [1]: a satellite constellation, ground control/monitoring networks and receivers. The satellite constellation includes satellites in orbit, which provide navigation ranging signals and navigation data messages for receivers. Therefore, GNSS constellation is the core of navigation satellite system, and the vulnerability of GNSS constellation is an important part of the vulnerability of GNSS. The vulnerability of GNSS can be mitigated by evaluating the vulnerability of GNSS constellation and improving GNSS constellation.

An assessment vulnerability method for GNSS constellation based on AHP-FCE is suggested in this article. Because the probability of more than 3 satellites failure is small [1], the vulnerability assessment model for GNSS constellation only concerns about the coverage performance of GNSS constellation in the case of 1 satellite failure, 2 satellites failure, and 3 satellites failure.

The paper is organized as follows: In the Sect. 27.2, AHP-FCE is briefly introduced. Section 27.3 suggests a vulnerability assessment model for GNSS constellation and provides the steps in which AHP-FCE is used to evaluate the vulnerability. Section 27.4 shows the vulnerability assessment results of COMPASS constellation and GPS constellation. Finally conclusions are drawn in Sect. 27.5.

27.2 AHP and FCE Method

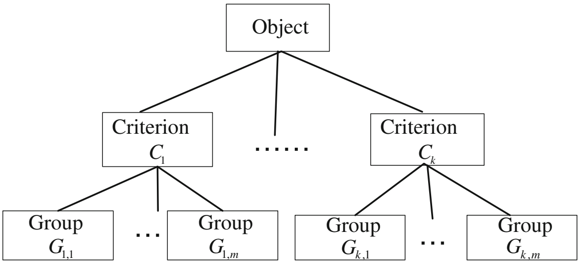
AHP-FCE assessment method is a combination of Analytical Hierarchy Process and Fuzzy Comprehensive Evaluation.

Analytic Hierarchy Process (AHP) is a multi-attribute decision-making method which can be used for scheme selection, evaluation, and decision-making [2]. The main steps of AHP method are as follows [3]:

1. Establish a hierarchy model

An analytic hierarchy model is constructed on the basis of an actual problem. In the hierarchy model, a complex problem is broken down into many elements (or criteria), and the elements are divided into groups in different layers according to their attributes. The established hierarchy model should be able to reflect intrinsic attributes and all internal relations of the evaluated object, as shown in Fig. 27.1.

Fig. 27.1 Hierarchy model



2. Establish judgment matrices

After establishing the hierarchy model and determining the relationship between various criterions of the object to be evaluated, the relative importance of different criterions in the same layer should be determined, and then the judgment matrices can be constructed by making use of digital scales.

3. Calculate the weight of criterions in the same layer

This step is a process to compute the relative weight of criterions in the same layer according to the judgment matrices. The relative weight can be calculated by calculating the eigenvalue and the eigenvector of judgment matrices.

4. Calculate the weight of criterions in different layers

The above step is repeated, and eigenvalues and eigenvectors of the judgment matrices are calculated along the layers. The relative weight of criterions can be calculated to make decisions.

The fuzzy comprehensive evaluation (FCE), which is based on fuzzy math, makes use of the principles of fuzzy relation synthesis to evaluate an object [4]. The main steps of FCE method are as follows:

1. Determine the evaluation criterions

It is needed to identify criterions which characterize the objects to be evaluated. The main criterions which reflect the objects to be evaluated can be selected according to the evaluating purpose.

2. Determine a comment set or evaluation grades

A comment set or evaluation grades can be determined for each of evaluation criterions.

3. Generate fuzzy matrices

The ranges of different criterions are different due to the different dimensions of criterions, so the values of criterions need to be processed as a normalized one. The criterions with different physical meanings are normalized to be a dimensionless value in the interval [0, 1]. Before starting fuzzy synthesis calculation, fuzzy relationship matrices are calculated according to the membership function.

4. Fuzzy synthesis calculation

The membership degree of evaluation grades of the evaluated object can be calculated on the basis of a fuzzy matrix R and a weight vector W.

The vector of fuzzy comprehensive evaluation result is defined as $S = (s_1, s_2, \ldots, s_n)$, and S can be calculated by the fuzzy matrix R and the weight vector W through the fuzzy operator. It can be expressed as:

$$S = W \circ R \tag{27.1}$$

where $\circ$ is the fuzzy operator symbol. Different operator symbols correspond to different fuzzy comprehensive evaluation models.

27.3 Vulnerability Assessment for GNSS Constellation

27.3.1 Vulnerability Assessment Criterions for GNSS Constellation

The vulnerability assessment for GNSS constellation needs to evaluate the coverage performance of the GNSS constellation in the case of 1, 2, and 3 satellites failure. Failed satellites in different orbital positions have different impacts on the coverage performance of GNSS constellation. The assessment criterions mainly reflect the mean impact on the coverage performance in the case of satellites failure.

The coverage performance of GNSS constellation is mainly reflected by a global coverage of GNSS constellations. A receiver needs to receive signals from at least four satellites for the positioning function; a receiver needs to receive signals from at least five satellites to detect whether there is an unacceptable positioning error, and needs to receive signals from at least six satellites to exclude the data of failed satellites from the navigation solution [1]. Therefore, if some satellites of GNSS constellation were failure, the receivers would not be able to receive enough navigation signals, which would affect the autonomous integrity monitoring function or the positioning function. Thus, the vulnerability assessment for GNSS constellation mainly concerns about the coverage of 4 satellites, 5 satellites, and 6 satellites.

When GNSS constellation loses n satellites, the mean coverage of at least k navigation satellites can be defined as $CL(n, k)$, where n is the number of failed satellites, and k is the minimum number of satellites which can be received by receivers in one day. For example, when 2 satellites are failure, the mean coverage of 5 satellites can be expressed as $CL(2, 5)$.

The number of received signals by receivers on the ground varies at different times. When the number of received signals is less than the required number, the corresponding function can't work. The global coverage that the minimum number of satellites received on the ground isn't less than k in one day is used in the vulnerability assessment for GNSS constellation.

27.3.2 Vulnerability Assessment Model for GNSS Constellation

The vulnerability assessment model for GNSS constellation can be established according to AHP-FCE. The vulnerability assessment model for GNSS constellation is a three-layer assessment model, and the assessment model is shown in Fig. 27.2.

The ultimate goal is to assess the vulnerability of GNSS constellation. The middle layer is the vulnerability of GNSS constellation in the case of 1 satellite failure, 2 satellites failure, and 3 satellites failure. The bottom layer is the coverage performance of GNSS constellation.

27.3.3 Relative Weight Calculation

The 1–9 scale method is used in the vulnerability assessment for GNSS constellation, its meaning is shown in Table 27.1 [3]:

After determining judgment matrices, the weight vectors are calculated by using the eigenvalue method, the basic steps are as follows [5]:

1. Each column of the matrix A is normalized: $\tilde{w}_{ij} = a_{ij} / \sum_{i=1}^{n} a_{ij}$
2. Summing $\tilde{w}_{ij}$ according to row: $\tilde{w}_i = \sum_{j=1}^{n} \tilde{w}_{ij}$
3. $\tilde{w}_i$ is normalized: $w_i = \tilde{w}_i / \sum_{i=1}^{n} \tilde{w}_{ij}$
4. The weight vector is $w = (w_1, w_2, \ldots, w_n)^T$.

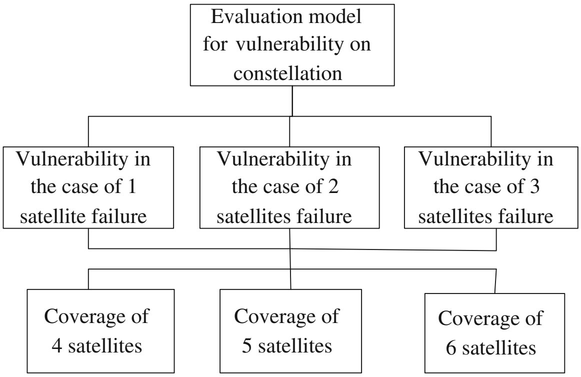


Fig. 27.2 Vulnerability assessment model

Table 27.1 Fundamental linguistic variables for pairwise comparisons

Scale	Definition
1	Equally important
3	Moderately more important
5	Strongly more important
7	Very strongly more important
9	Exceedingly more important
2, 4, 6, 8	Intermediate preferences
Reciprocal	If the importance of criterion i with respect to criterion j is a_{ij}, the importance of criterion j with respect to criterion i is $a_{ji} = 1/a_{ij}$

27.3.4 Generate Fuzzy Matrices

Fuzzy matrices are generated on the basis of the global coverage of GNSS constellation. Triangular fuzzy numbers are selected as membership functions in the process of assessment vulnerability for GNSS constellation. The comment set includes five grades: excellent, good, middle, poor, worst. The membership functions are as follows [5]:

$$S_1(x) = \begin{cases} (m_2 - x)/m_2 & 0 \le x \le m_2 \\ 0 & m_2 \le x \le 1 \end{cases} \tag{27.2}$$

$$S_i(x) = \begin{cases} 0 & 0 \le m_{i-1} \\ (x - m_{i-1})/(m_i - m_{i-1}) & m_{i-1} \le x \le m_i \\ (m_{i+1} + x)/(m_{i+1} - m_i) & m_i \le x \le m_{i+1} \\ 0 & m_{i+1} \le x \le 1 \end{cases} \tag{27.3}$$

$$S_5(x) = \begin{cases} 0 & 0 \le m_4 \\ (x - m_4)/(1 - m_4) & m_4 \le x \le 1 \end{cases} \tag{27.4}$$

where $i = 2, 3, 4$, $m_1 = 0$, $m_2 = 0.25$, $m_3 = 0.5$, $m_4 = 0.75$, $m_5 = 1$.

The fuzzy relation matrix of n satellites failure can be described as follow:

$$R_n = \begin{bmatrix} S_{n,4} & S_{n,5} & S_{n,6} \end{bmatrix}^T \tag{27.5}$$

where $S_{n,4}$, $S_{n,5}$, $S_{n,6}$, are respectively the membership degrees of the coverage performance of 4 satellites, 5 satellites, 6 satellites in the case of n satellites failure.

27.3.5 Fuzzy Synthesis Calculation

The vulnerability assessment model for GNSS constellation is a three-layer assessment model. Therefore, 2 times of synthesis operations are needed to calculate the final assessment result.

The fuzzy relationship matrix of 1 satellite failure, 2 satellites failure and 3 satellites failure can be respectively expressed as R_1, R_2, R_3. The weights of $S_{n,4}$, $S_{n,5}$, $S_{n,6}$ are respectively expressed as $w_{n,4}$, $w_{n,5}$, $w_{n,6}$. After finishing the fuzzy synthesis calculation, the assessment results in the case of n satellites failure can be formulated as follow:

$$r_n = \begin{bmatrix} w_{n,4} & w_{n,5} & w_{n,6} \end{bmatrix} R_n \tag{27.6}$$

The new fuzzy relation matrix consists of r_1, r_2, r_3. The weights of vulnerability caused by 1 satellite failure, 2 satellites failure, and 3 satellites failure are respectively described as w_1, w_2, and w_3. After the second fuzzy synthesis calculation, the final assessment results of GNSS constellation can be expressed as follow:

$$r = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \tag{27.7}$$

27.4 Vulnerability Assessment for COMPASS Constellation and GPS Constellation

COMPASS constellation is designed to have 27 medium earth orbit (MEO) satellites, 5 geostationary earth orbit (GEO) satellites and 3 inclined geosynchronous orbit (IGSO) satellites [6]. In the simulation, 27 MEO satellites in COMPASS constellation are assumed to locate in three orbital planes as Galileo constellation. GPS constellation consists of 31 satellites and these satellites locate in six orbital planes which have about five satellites.

When the shield angle is 5°, and 1, 2, 3 satellites are failure, the coverage performance of COMPASS constellation and GPS constellation can be simulated by STK and the simulation results are shown in Tables 27.2 and 27.3:

Table 27.2 The mean coverage of COMPASS constellation

Coverage	Satellites failure		
	1 satellite failure	2 satellite failure	3 satellite failure
At least 4 satellites	1	1	0.999
At least 5 satellites	1	0.994	0.981
At least 6 satellites	0.948	0.908	0.863

Table 27.3 The mean coverage of GPS constellation

Coverage	Satellites failure		
	1 satellite failure	2 satellite failure	3 satellite failure
At least 4 satellites	1	1	0.999
At least 5 satellites	0.998	0.978	0.977
At least 6 satellites	0.929	0.820	0.763

27.4.1 Calculate Relative Weights

It is known that the main function of GNSS is the positioning function which needs at least four navigation signals. Therefore, the coverage performance of four satellites is more important than the coverage performance of five satellites. Similarly, the coverage performance of five satellites is more important than the coverage performance of six satellites. The relative importance is shown in Table 27.4:

The weight vector can be obtained by the eigenvalue method:

$$\mathrm{w} = [0.6333,\ 0.2605,\ 0.1062]$$

In the process of the vulnerability analysis, the probability of 1 satellite failure is higher than the probability of 2 satellites failure, and the probability of 2 satellites failure is higher than the probability of 3 satellites failure, so the relative importance of 1 satellite failure, 2 satellites failure, and 3 satellites failure is shown in Table 27.5:

The weight vector can be obtained by the eigenvalue method:

$$\mathrm{w} = [0.7235,\ 0.1932,\ 0.0833]$$

27.4.2 Generate Fuzzy Matrices of COMPASS Constellation

The fuzzy matrices of COMPASS constellation can be calculated by the fuzzy membership function mentioned above.

When 1 satellite is failure, the fuzzy relation matrix can be described as follow:

Table 27.4 The relative importance of criterions

	$CL(n,4)$	$CL(n,5)$	$CL(n,6)$
$CL(n,4)$	1	3	5
$CL(n,5)$	1/3	1	3
$CL(n,6)$	1/5	1/3	1

Table 27.5 The relative importance of satellites failure

	1 satellite failure	2 satellites failure	3 satellites failure
1 satellite failure	1	5	7
2 satellites failure	1/5	1	3
3 satellites failure	1/7	1/3	1

$$R_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.208 & 0.792 \end{bmatrix}$$

When 2 satellites are failure, the fuzzy relation matrix can be described as follow:

$$R_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.024 & 0.976 \\ 0 & 0 & 0 & 0.368 & 0.632 \end{bmatrix}$$

When 3 satellites are failure, the fuzzy relation matrix can be described as follow:

$$R_3 = \begin{bmatrix} 0 & 0 & 0 & 0.004 & 0.996 \\ 0 & 0 & 0 & 0.076 & 0.924 \\ 0 & 0 & 0 & 0.548 & 0.452 \end{bmatrix}$$

Similarly, the fuzzy matrices of GPS constellation can also be calculated by the fuzzy membership function.

27.4.3 Fuzzy Synthesis Calculation

The weight vector $[w_{\mathrm{n},4}, w_{\mathrm{n},5}, w_{\mathrm{n},6}] = [0.6333, 0.2605, 0.1062]$ is applied during fuzzy synthesis calculation. According to the fuzzy synthesis steps, the results of constellation vulnerability which are caused by 1, 2, 3 satellites failure can be calculated in the first fuzzy synthesis calculation.

When 1 satellite is failure, the fuzzy synthesis result is shown as follow:

$$r_1 = \begin{bmatrix} w_{n,4} & w_{n,5} & w_{n,6} \end{bmatrix} \cdot R_1 = \begin{bmatrix} 0 & 0 & 0 & 0.0221 & 0.9779 \end{bmatrix}$$

When 2 satellites are failure, the fuzzy synthesis result is shown as follow:

$$r_2 = \begin{bmatrix} w_{n,4} & w_{n,5} & w_{n,6} \end{bmatrix} \cdot R_2 = \begin{bmatrix} 0 & 0 & 0 & 0.0453 & 0.9547 \end{bmatrix}$$

When 3 satellites are failure, the fuzzy synthesis result is shown as follow:

$$r_3 = \begin{bmatrix} w_{n,4} & w_{n,5} & w_{n,6} \end{bmatrix} \cdot R_3 = \begin{bmatrix} 0 & 0 & 0 & 0.0805 & 0.9195 \end{bmatrix}$$

These calculation results can be combined as the input of the fuzzy synthesis calculation in the second time. The fuzzy relationship matrix can be established as follow:

$$R = \begin{bmatrix} 0 & 0 & 0 & 0.0221 & 0.9779 \\ 0 & 0 & 0 & 0.0453 & 0.9547 \\ 0 & 0 & 0 & 0.0805 & 0.9195 \end{bmatrix}$$

The weight vector w = [0.7235, 0.1932, 0.0833] is used during the second fuzzy synthesis calculation, and the assessment result is as follow:

$$r = w \cdot R = \begin{bmatrix} 0 & 0 & 0 & 0.0314 & 0.9686 \end{bmatrix}$$

The membership degrees of the vulnerability of COMPASS constellation in five grades are respectively 0, 0, 0, 0.0314, and 0.9686. According to the principle of maximum membership degree, the comment of the vulnerability of COMPASS constellation is "excellent".

Similarly, the membership degrees of the vulnerability of GPS constellation in five grades are respectively 0.0000, 0.0000, 0.0000, 0.0529, and 0.9471. According to the principle of maximum membership degree, the comment of the vulnerability of GPS constellation is also "excellent".

Although the comments of COMPASS constellation and GPS constellation are both "excellent", the membership degree of GPS constellation is 0.9471, and the membership degree of COMPASS constellation is 0.9686. It means that GPS constellation is slightly more vulnerable than COMPASS constellation.

27.5 Conclusions

GNSS plays an important role in the national defense and economy, the vulnerability assessment for GNSS is becoming more and more important. GNSS constellation is the core of the navigation satellite system, and the vulnerability of GNSS constellation is an important part of the vulnerability of GNSS.

A vulnerability assessment method for GNSS constellation based on AHP-FCE is suggested in this paper. The probability of more than 3 satellites failure is lower, so the vulnerability assessment model for GNSS constellation only concerns about 1 satellite failure, 2 satellites failure, and 3 satellites failure. Failed satellites in different orbital positions have different impacts on the global coverage performance of GNSS constellation, so the assessment criterions need to reflect the mean impact on the coverage performance in the case of satellites failure. Although the vulnerability assessment model for GNSS constellation only includes the situation that 1, 2 and 3 satellites are failure in this paper, it's very easy to add criterions reflecting 4 or more satellites failure into the assessment model. The assessment model mainly concerns about the impacts on the coverage performance of GNSS constellations in the case of satellites failure, which provides a new way to evaluate the vulnerability of GNSS constellations.

Acknowledgments This work was supported by the National High Technology Research and Development Program of China (863 Program). The fund number is 2011AA120503.

References

1. Kaplan ED, Hegarty CJ (2008) Understanding GPS: principles and applications, 2nd edn. Publishing House of Electronics Industry, Beijing (In Chinese)
2. Vaidya OS, Kumar S (2006) Analytic hierarchy process: an overview of applications. Eur J Oper Res 169(1):P1–P29
3. Saaty L (1980) The analytic hierarchy process. McGraw-Hill, New York
4. Zhu M (2008) Evaluating destination competitiveness based on integrated fuzzy AHP and FCE. In: Proceedings of the seventh international conference on information and management sciences, Urumchi, China, pp 289–294
5. Haitao Yang (2007) Research on methods of constellation satellite communication system effectiveness evaluation. National University of Defense Technology, Changsha
6. China Satellite Navigation Office (2012) BeiDou navigation satellite system signal in space interface control document open service signal B1I (1.0)